

Intensity-dependent pion-nucleon coupling and the Wróblewski relation

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Abstract

We propose an intensity-dependent pion-nucleon coupling Hamiltonian within a unitary multiparticle-production model of the Auerbach-Avin-Blankenbecler-Sugar (AABS) type in which the pion field is represented by the thermal-density matrix. Using this Hamiltonian, we explain the appearance of the negative-binomial (NB) distribution for pions and the well-known empirical relation, the so-called Wróblewski relation, in which the dispersion D of the pion-multiplicity distribution is linearly related to the average multiplicity $\langle n \rangle$: $D = A \langle n \rangle + B$, with the coefficient A related to the vacuum energy of the pion field and < 1 . The Hamiltonian of our model is expressed linearly in terms of the generators of the $SU(1,1)$ group. We also find the generating function for the pion field. All higher-order thermal moments can be calculated from this function. At $T = 0$ it reduces to the generating function of the NB distribution.

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1 Introduction

During the last years a considerable amount of experimental information has been accumulated on multiplicity distributions of charged particles produced in pp and $p\bar{p}$ collisions in the centre- of-mass energy range from 10GeV to 1800GeV . Measurements in the regime of several hundred GeV [1] have shown the violation of the Koba-Nielsen-Olesen (KNO) scaling [2], which was previously observed in the ISR c.m.energy range from 11 to 63 GeV [3]. The violation of the KNO scaling is characterized by an enhancement of high-multiplicity events leading to a broadening of the multiplicity distribution with energy.

The shape of the multiplicity distribution may be described either by its C moments, $C_q = \langle n^q \rangle / \langle n \rangle^q$, or by its central moments (higher-order dispersions), $D_q = \langle (n - \langle n \rangle)^q \rangle^{1/q}$, $q = 2, 3, \dots$. The exact KNO scaling implies that all C_q moments are energy independent. Only at energies below 100GeV do the C moments appear to be energy independent. It can also be shown [4] that the KNO scaling leads to a generalized Wróblewski relation [5]

$$D_q = A_q \langle n \rangle - B_q, \quad (1)$$

with the energy-independent coefficients A_q and B_q . The pp and $p\bar{p}$ inelastic data below 100GeV also show the linear dependence of the dispersion on the average number of charged particles, but with the coefficients A_q and B_q that are approximately equal within errors.

The fact that the dispersion of the multiplicity distribution grows linearly with $\langle n \rangle$ implies that the elementary Poisson distribution resulting from the independent emission of particles is ruled out.

The total multiplicity distribution P_n of charged particles for a wide range of energies ($22 - 900\text{GeV}$) is found to be well described by a negative-binomial (NB)

distribution [1,6] that belongs to a large class of compound Poisson distributions [7]. The compound Poisson distribution is fully determined by its generating function $G(z)$ of the form

$$G(z) = e^{\bar{N}[g(z)-1]} = \sum z^n P_n, \quad (2)$$

where \bar{N} is the average number of N independently produced clusters or clans (Poisson distributed) [8]. Each of these subsequently decays according to the probability distribution corresponding to the generating function $g(z)$. The NB distribution is obtained by choosing $g(z) = \ln(1 - pz)/\ln(1 - p)$, with $p = (1 - \langle n \rangle/k)^{-1}$. It is a two-step process [8] with two free parameters: the average number of charged particles $\langle n \rangle$ and the parameter k which affects the shape (width) of the distribution. The parameter k is also related to the dispersion $D = D_2$ by the relation

$$\left(\frac{D}{\langle n \rangle}\right)^2 = \frac{1}{k} + \frac{1}{\langle n \rangle}, \quad (3)$$

so that the observed broadening of the normalized multiplicity distribution with increasing energy implies a decrease of the parameter k with energy. The KNO scaling requires constant k .

Although the NB distribution gives information on the structure of correlation functions in multiparticle production, the question still remains whether its clan-structure interpretation is simply a new parametrization of the data or has a deeper physical insight [9]. Measurements of multiplicity distributions in $p\bar{p}$ collisions at TeV energies [10] have recently shown that their shape is clearly different from that of the NB distribution. The distributions display the so-called medium-multiplicity "shoulder", with a shape qualitatively similar to that of the UA5 $900GeV$ and UA1 distributions [11]. A satisfactory explanation of this effect is still lacking [12].

In this paper we propose another approach to multiplicity distributions based on a unitary eikonal model with a pion-field thermal-density operator given in terms

of an effective intensity-dependent pion-nucleon coupling Hamiltonian. We assume that the system of produced hadronic matter (pions) is in thermal equilibrium at the temperature T immediately after the collision.

The paper is organized as follows. In Sect.2 we explain the basic ideas of our unitary eikonal model with a pion-field thermal-density operator. A discussion of the Wróblewski relation is presented in Sect. 3. Finally, in Sect. 4 we draw conclusions and make remarks on the possible extension of the model to include two-pion correlations in the effective pion-nucleon Hamiltonian.

2 Description of the model

At present accelerator energies the number of secondary particles (mostly pions) produced in hadron-hadron collisions is large enough, so that the statistical approach to particle production becomes reasonable . Most of the properties of pions produced in high-energy hadron-hadron collisions can be expressed simply in terms of a pion-field density operator. We neglect difficulties associated with isospin and only consider the production of isoscalar "pions". We expect that in high-energy collisions most of the pions are produced in the central region. In this region the energy - momentum conservation has a minor effect if the transverse momenta of the pions are limited by the dynamics and their rapidities restricted to the range $|y| < Y$, where $Y = \ln(s/m^2)$ is the relative rapidity of the colliding particles.

A. The AABS model

A long time ago Auerbach, Avin, Blankenbecler, and Sugar [13] presented a class of models (AABS models) for which the scattering operator satisfied the exact s-channel unitarity at high energies. In these models, the incident hadrons propagate through the interaction region, without making significant changes in their longitudinal momenta (leading-particle effect). Only the part $W = K\sqrt{s}$ of the total c.m. energy \sqrt{s} in every concrete event is available for particle production, where K is the inelasticity: $0 \leq K \leq 1$.

In the AABS type of models, the scattering operator \hat{S} is diagonal in the rapidity difference Y and in the relative impact parameter \vec{B} of the two incident hadrons. The initial-state vector for the pion field is $\hat{S}(Y, \vec{B}) | 0 \rangle$, where the vacuum state $| 0 \rangle$ for pions is in fact a state containing two incident hadrons.

The n -pion production amplitude for $n \geq 1$ is given by

$$iT_n(Y, \vec{B}; k_1 \dots k_n) = 2s \langle k_1 \dots k_n | \hat{S}(Y, \vec{B}) | 0 \rangle. \quad (4)$$

We write the square of the n -pion production amplitude in the form

$$|T_n(Y, \vec{B}; k_1 \dots k_n)|^2 = 4s^2 \text{Tr}\{\rho(Y, \vec{B}) | k_1 \dots k_n\rangle \langle k_1 \dots k_n | \}, \quad (5)$$

where the pion-density operator $\rho(Y, \vec{B})$ is defined as

$$\rho(Y, \vec{B}) = \hat{S}(Y, \vec{B}) | 0\rangle \langle 0 | \hat{S}^\dagger(Y, \vec{B}). \quad (6)$$

The square of the elastic scattering amplitude is then the matrix element of $\rho(Y, \vec{B})$ between the states with no pions, i.e., $\langle 0 | \rho(Y, \vec{B}) | 0\rangle$. In terms of the pion-number operator

$$\hat{N} = \sum_k a_k^\dagger a_k = \sum_k \hat{N}_k, \quad k \equiv (\omega_k, \vec{k}), \quad (7)$$

the square of the S -matrix element when no pions are emitted can also be written in the form

$$| \langle 0 | \hat{S}(Y, \vec{B}) | 0\rangle |^2 = \text{Tr}\{\rho(Y, \vec{B}) : e^{-\hat{N}} : \} = e^{-\Omega(Y, \vec{B})}. \quad (8)$$

Here $::$ indicates the operation of normal ordering and $\Omega(Y, \vec{B})$ is the usual eikonal function (or the opacity function) of the geometrical model [14]. The connection with the inelastic cross section and the exclusive cross section for production of n pions is then

$$\sigma_{inel}(Y, \vec{B}) = 1 - e^{-\Omega(Y, \vec{B})}, \quad (9)$$

and for $n \geq 1$, it is

$$\sigma_n(Y, \vec{B}) = \text{Tr}\{\rho(Y, \vec{B}) : \frac{\hat{N}^n}{n!} e^{-\hat{N}} : \}. \quad (10)$$

In terms of a normalized pion-multiplicity distribution at each impact parameter, $P_n(Y, \vec{B}) = \sigma_n(Y, \vec{B})/\sigma_{inel}(Y, \vec{B})$, the observed complete multiplicity distribution $P_n(Y)$ is obtained by summing $P_n(Y, \vec{B})$ over all impact parameters \vec{B} with the weight function $Q(Y, \vec{B}) = \sigma_{inel}(Y, \vec{B})/\sigma_{inel}(Y)$, i.e.,

$$P_n(Y) = \int d^2B Q(Y, \vec{B}) P_n(Y, \vec{B}). \quad (11)$$

The first-order moment of $P_n(Y)$ is the average multiplicity

$$\langle n \rangle = \sum n P_n(Y) = \int d^2 B Q(Y, \vec{B}) \bar{n}(Y, \vec{B}). \quad (12)$$

The higher-order moments of $P_n(Y)$ give information on the dynamical fluctuations from $\langle n \rangle$ and also on the multiparticle correlations. All these higher-order moments can be obtained from the pion-generating function

$$G(z) = \sum z^n P_n(Y) = \int d^2 B Q(Y, \vec{B}) G(Y, \vec{B}; z), \quad (13)$$

by differentiation where

$$G(Y, \vec{B}; z) = \text{Tr}\{\rho(Y, \vec{B}) z^{\hat{N}}\} \quad (14)$$

is the pion-generating function in B -space. Thus the normalized factorial moments F_q are

$$F_q = \frac{\langle n(n-1) \dots (n-q+1) \rangle}{\langle n \rangle^q} = \langle n \rangle^{-q} \frac{d^q G(1)}{dz^q} \quad (15)$$

and the normalized cumulant moments K_q are

$$K_q = \langle n \rangle^{-q} \frac{d^q \ln G(1)}{dz^q}. \quad (16)$$

These moments are related to each other by the formula

$$F_q = \sum_{l=0}^{q-1} \binom{q-1}{l} K_{q-l} F_l. \quad (17)$$

For the Poisson distribution, all the normalized factorial moments are identically equal to 1 and all cumulants vanish for $q > 1$.

We are concerned here mostly with the $q = 2$ moments, which are directly related to the dispersion D :

$$F_2 = K_2 + 1 = \left(\frac{D}{\langle n \rangle}\right)^2 + 1 - \frac{1}{\langle n \rangle}. \quad (18)$$

B. Thermal-density operator for the pion field

The operator $|0\rangle\langle 0|$ appearing in the definition of $\rho(Y, \vec{B})$ represents the density operator $\rho(vac)$ for the pion-field vacuum state. It can also be considered as describing the pion system in thermal equilibrium at the temperature $T = 0$. The density operator for a pion field in thermal equilibrium at the temperature T is then

$$\rho_T = \frac{1}{Z} e^{-\beta H_0}, \quad \beta = \frac{1}{k_B T}, \quad (19)$$

where

$$H_0 = \sum_k \omega_k (a_k^\dagger a_k + \lambda), \quad (20)$$

$$\ln Z = -\beta \lambda \sum_k \omega_k - \sum_k \ln(1 - e^{-\beta \omega_k}).$$

The quantity $\lambda \sum_k \omega_k$ represents the lowest possible energy of the pion system at the temperature $T = 0$. The "zero-point energy" corresponds to $\lambda = \frac{1}{2}$. If the pion energies $\omega_k = \sqrt{\vec{k}^2 + m_\pi^2}$ are closely spaced, the summation over k is replaced by an integral: $\sum_k \rightarrow \int d^3k / 2\omega_k$.

Note that $\rho(vac) = \rho_{T=0}$. The mean number of thermal (chaotic) pions is

$$\begin{aligned} \bar{n}_T &= \sum_k \frac{1}{e^{\beta \omega_k} - 1} \\ &= \sum_k \bar{n}_{Tk}. \end{aligned} \quad (21)$$

The transformed thermal-density operator $\rho_T(Y, \vec{B})$ is now

$$\begin{aligned} \rho_T(Y, \vec{B}) &= \hat{S}(Y, \vec{B}) \rho_T \hat{S}^\dagger(Y, \vec{B}) \\ &= \frac{1}{Z} e^{-\beta H(Y, \vec{B})}, \end{aligned} \quad (22)$$

where

$$H(Y, \vec{B}) = \hat{S}(Y, \vec{B}) H_0 \hat{S}^\dagger(Y, \vec{B}) \quad (23)$$

is regarded as an effective Hamiltonian describing the pion system in the presence of two leading particles (nucleons). Taking into account an old observation of Golab-Meyer and Ruijgrok [15] that the Wróblewski relation can be satisfied for all energies if the square of the pion-nucleon coupling constant increases linearly with the mean number of pions $\langle n \rangle$, we propose the following form of the effective pion-nucleon intensity-dependent coupling Hamiltonian:

$$\begin{aligned} H(Y, \vec{B}) &= \sum_k [\epsilon_k(Y, \vec{B})(N_k + \lambda) + g_k(Y, \vec{B})(a_k \sqrt{N_k + 2\lambda - 1} + h.c.)] \quad (24) \\ &= \sum_k H_k(Y, \vec{B}), \end{aligned}$$

where $\epsilon_k^2(Y, \vec{B}) = \omega_k^2 + 4g_k^2(Y, \vec{B})$. The interaction part of the Hamiltonian H_k for the k mode is no longer linear in the pion-field variables a_k and represents an intensity-dependent coupling [16]. It is also easy to see that the operators

$$\begin{aligned} K_0(k) &= N_k + \lambda, \\ K_-(k) &= a_k \sqrt{N_k + 2\lambda - 1}, \\ K_+(k) &= \sqrt{N_k + 2\lambda - 1} a_k^\dagger \end{aligned} \quad (25)$$

form the standard Holstein-Primakoff [17] realizations of the $su(1, 1)$ Lie algebra, the Casimir operator of which is

$$\hat{C}_k = K_0^2(k) - \frac{1}{2}[K_+(k)K_-(k) + K_-(k)K_+(k)] = \lambda(\lambda - 1)\hat{I}_k. \quad (26)$$

The Hamiltonian $H_k(Y, \vec{B}) \equiv H_k$ is thus a linear combination of the generators of the $SU(1, 1)$ group:

$$H_k = \epsilon_k K_0(k) + g_k [K_+(k) + K_-(k)]. \quad (27)$$

The corresponding S-matrix which diagonalizes the Hamiltonian $H(Y, \vec{B})$ is

$$\hat{S}(Y, \vec{B}) = \prod_k \hat{S}_k(Y, \vec{B}), \quad (28)$$

where

$$\hat{S}_k(Y, \vec{B}) = \exp\{-\theta_k(Y, \vec{B})[K_+(k) - K_-(k)]\}, \quad (29)$$

with

$$th\theta_k(Y, \vec{B}) = \frac{2g_k(Y, \vec{B})}{\epsilon_k(Y, \vec{B})}. \quad (30)$$

Since the dependence on the variables Y, \vec{B} is contained only in the hyperbolic angle $\theta_k(Y, \vec{B})$, we shall from now on assume this dependence whenever we write θ_k .

It is easy to see that the initial-state vector for the pion field, $\hat{S}(Y, \vec{B}) | 0 \rangle$, factorizes

$$\hat{S}(Y, \vec{B}) | 0 \rangle = \prod_k (\hat{S}_k(Y, \vec{B}) | 0_k \rangle), \quad (31)$$

with

$$\begin{aligned} \hat{S}_k(Y, \vec{B}) | 0_k \rangle &= (1 - th^2\theta_k)^\lambda \sum_{n_k} (-th\theta_k)^{n_k} \left(\frac{\Gamma(n_k + 2\lambda)}{n_k! \Gamma(2\lambda)} \right)^{1/2} | n_k \rangle \\ &= | \theta_k \rangle. \end{aligned} \quad (32)$$

where $| n_k \rangle = (n_k!)^{-1/2} (a_k^\dagger)^{n_k} | 0_k \rangle$. In the same way we find that the pion thermal-density operator $\rho_T(Y, \vec{B})$ is also factorized as

$$\rho_T(Y, \vec{B}) = \prod_k \rho_T(\theta_k), \quad (33)$$

with

$$\rho_T(\theta_k) = \frac{1}{Z_k} \sum_{n_k} e^{-\beta\omega_k(n_k + \lambda)} | n_k, \theta_k \rangle \langle n_k, \theta_k |, \quad (34)$$

where $| n_k, \theta_k \rangle = \hat{S}_k(Y, \vec{B}) | n_k \rangle$. Note that $| n_k, \theta_k \rangle$ form a complete orthonormal set of eigenvectors of the k -mode Hamiltonian H_k , i.e.,

$$H_k | n_k, \theta_k \rangle = \omega_k(n_k + \lambda) | n_k, \theta_k \rangle. \quad (35)$$

3 Pion-generating function and its moments

The average multiplicity $\bar{n}_T(Y, \vec{B})$, the dispersion $d_T^2(Y, \vec{B})$, and all higher-order moments

$$\bar{n}_T^q(Y, \vec{B}) = Tr\{\rho_T(Y, \vec{B})\hat{N}^q\}, \quad q = 1, 2, \dots \quad (36)$$

at the temperature T in B space can be obtained from the pion-generating function

$$G_T(Y, \vec{B}; z) = \prod_k G_T(\theta_k; z) \quad (37)$$

by differentiation, where

$$G_T(\theta_k; z) = Tr\{\rho_T(\theta_k) z^{\hat{N}_k}\}. \quad (38)$$

After performing a certain amount of straightforward algebraic manipulations, we arrive [18] at the following expression for the pion-generating function $G_T(\theta_k; z)$:

$$G_T(\theta_k; z) = G_0(\theta_k; z)(1 - e^{-\beta\omega_k})2^{2\lambda-1}R_k^{-1}(1 + y_k + R_k)^{1-2\lambda}, \quad (39)$$

where

$$\begin{aligned} R_k &= \sqrt{1 - 2x_k y_k + y_k^2}, \\ x_k &= \frac{z + (1 - z)^2 sh^2(\theta_k) ch^2(\theta_k)}{z - (1 - z)^2 sh^2(\theta_k) ch^2(\theta_k)}, \\ y_k &= e^{-\beta\omega_k} \frac{z - (1 - z) sh^2(\theta_k)}{1 + (1 - z)^2 sh^2(\theta_k)}. \end{aligned} \quad (40)$$

and $G_0(\theta_k; z)$ denotes the pion-generating function at the temperature $T = 0$:

$$G_0(\theta_k; z) = [1 + (1 - z) sh^2(\theta_k)]^{-2\lambda}. \quad (41)$$

We observe that G_0 is exactly the generating function of the NB distribution with a constant shape parameter 2λ , and the average number of k-mode pions is equal to

$$\bar{n}(\theta_k) = 2\lambda sh^2(\theta_k). \quad (42)$$

The vacuum value of the k -mode thermal-density operator $\rho_T(\theta_k)$ is used to obtain the k -mode thermal eikonal function $\Omega_T(\theta_k)$

$$\begin{aligned}\langle 0_k | \rho_T(\theta_k) | 0_k \rangle &= e^{-\Omega_T(\theta_k)} \\ &= (1 - e^{-\beta\omega_k})G_0(\theta_k; e^{-\beta\omega_k}).\end{aligned}\tag{43}$$

The total eikonal function is $\Omega_T(Y, \vec{B}) = \sum_k \Omega_T(\theta_k)$.

For the k -mode pion field in B space at the temperature T , we find the following average number and the dispersion:

$$\begin{aligned}\bar{n}_T(\theta_k) &= \bar{n}(\theta_k) + \bar{n}_{Tk} + \frac{1}{\lambda}\bar{n}(\theta_k)\bar{n}_{Tk}, \\ d_T^2(\theta_k) &= d_{Tk}^2 + d^2(\theta_k)[1 + \frac{2\lambda - 3}{\lambda}\bar{n}_{Tk} + \frac{4}{\lambda}\bar{n}_{Tk}^2],\end{aligned}\tag{44}$$

where

$$\begin{aligned}d_{Tk}^2 &= \bar{n}_{Tk}^2 + \bar{n}_{Tk}, \\ d^2(\theta_k) &= \frac{1}{2\lambda}\bar{n}^2(\theta_k) + \bar{n}(\theta_k).\end{aligned}\tag{45}$$

Two limiting cases are of interest namely, $T = 0$ and $T \rightarrow \infty$. For the $T = 0$ case, we find

$$\frac{d^2(\theta_k)}{\bar{n}^2(\theta_k)} = \frac{1}{2\lambda} + \frac{1}{\bar{n}(\theta_k)},\tag{46}$$

as it is to be expected from the NB distribution (Eq. 3). However, our interpretation of this result is quite different. In our case, the parameter λ is connected with the vacuum energy of the pion field in Eq. 20, and has nothing to do with either the number of pion sources or the number of clans. It is a real and positive constant labeling the positive discrete class of a unitary irreducible representation of $SU(1, 1)$, which is a dynamical symmetry group of our system Hamiltonian. It is important to observe that pions in the k mode are distributed according to the NB distribution

with a constant shape parameter 2λ . The Wróblewski relation

$$d(\theta_k) = A\bar{n}(\theta_k) + B \quad (47)$$

is obtained with energy-independent coefficients $A = (2\lambda)^{-1/2}$ and $B = (\lambda/2)^{1/2}$. If $\lambda > 1/2$, we have $A < 1$.

The contribution from all the k modes gives

$$\frac{d^2(Y, \vec{B})}{\bar{n}^2(Y, \vec{B})} = \frac{1}{2\lambda} \sum_k p^2(\theta_k) + \frac{1}{\bar{n}(Y, \vec{B})}, \quad (48)$$

where $p(\theta_k) = \bar{n}(\theta_k)/\bar{n}(Y, \vec{B})$. In this case, the coefficient A in the Wróblewski relation becomes energy dependent and is of the form

$$A(Y, \vec{B}) = \left[\frac{1}{2\lambda} \sum_k p^2(\theta_k) \right]^{1/2}. \quad (49)$$

Since $\sum_k p(\theta_k) = 1$ and all $p(\theta_k)$ are positive functions of θ_k , the sum $\sum_k p^2(\theta_k)$ is always smaller than one. Therefore, $A(Y, \vec{B}) < 1$ if $\lambda > 1/2$.

Finally, the summation over all impact parameters gives

$$\left(\frac{D}{\langle n \rangle} \right)^2 = \int d^2 B Q(Y, \vec{B}) [(A^2(Y, \vec{B}) + 1) \left(\frac{\bar{n}(Y, \vec{B})}{\langle n \rangle} \right)^2 - 1] + \frac{1}{\langle n \rangle}. \quad (50)$$

This expression, when combined with our preceding analysis suggests that the coefficient A in the Wróblewski relation should be energy dependent and smaller than one.

For the temperature T going to infinity we obtain

$$\begin{aligned} \frac{d_T^2(\theta_k)}{\bar{n}_{T_k}^2(\theta_k)} \Big|_{T \rightarrow \infty} &= 2 - \left(1 + \frac{\bar{n}(\theta_k)}{\lambda} \right)^{-2} \\ &= 1 + t h^2(2\theta_k). \end{aligned} \quad (51)$$

This result shows that at very high temperature the distribution of pions will become chaotic if θ_k is very small. This will happen when $\omega_k \gg g_k(Y, \vec{B})$.

4 Conclusions

In this paper we have proposed an intensity-dependent pion-nucleon coupling Hamiltonian with $SU(1, 1)$ dynamical symmetry, within a multiparticle-production model of the AABS type in which the k mode pion field is represented by the thermal-density operator. We have shown that this Hamiltonian explains in a natural way the appearance of the NB multiplicity distribution for pions in impact-parameter space. The shape parameter of the NB distribution is related to the vacuum energy of the pion field at the temperature $T = 0$.

The Wróblewski relation is obtained with the coefficient A that is energy dependent and smaller than one if the vacuum energy of the pion field is larger than "zero-point energy" corresponding to $\lambda = 1/2$.

For $T \neq 0$, we have found a pion-generating function that may be used for obtaining all higher-order moments of the pion field.

In our model, the k modes of the pion field are statistically independent and are described by the factorized thermal-density operator. Correlations between different k modes are absent and, at this stage, our model cannot describe the emission of resonances. However, this can be remedied by adding a mode-mode interacting part to the total Hamiltonian $H(Y, \vec{B})$, e.g., $\sum_{k,k'} [A_{k,k'}(Y, \vec{B}) a_k a_{k'} + h.c.]$ [19]. We hope to treat this case elsewhere.

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